Published by Institute of Physics Publishing for SISSA/ISAS

Received: *February 14, 2002*
Accepted: *February 20, 2002*

Higher-order corrections to QCD factorization in $B \to \pi K, \pi\pi$ decays

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Abstract: The renormalon calculus is used to calculate the terms of order $\beta_0^{n-1}\alpha_s^n$ in the perturbative expansions of the Wilson coefficients and hard-scattering kernels entering the QCD factorization formula for hadronic B-meson decays into two light pseudoscalar mesons. The asymptotic behavior of the expansions is analyzed, and a minimal model of power corrections arising from soft "non-factorizable" gluon exchange to the $B \to \pi K, \pi\pi$ decay amplitudes is obtained, which takes into account the structure of the leading and subleading infrared renormalon singularities. Whereas the resulting power corrections are generally very small, some of the strong-interaction phases of the hard-scattering kernels receive sizeable two-loop corrections. The implications of these findings on CP asymmetries and branching ratios are investigated.

Keywords: B-Physics, Renormalization Regularization and Renormalons, NLO Computations.

Contents

1. Introduction

A theoretical understanding of hadronic B decays is necessary to extract CKM matrix elements and CP-violating weak phases from data taken at the B factories. The formalism of QCD factorization developed in [1, 2, 3] has shown that two-body hadronic B decays simplify considerably in the heavy-quark limit. In $B \to \pi K$ decays, for example, QCD factorization states that the matrix elements of the local operators Q_i appearing in the effective weak hamiltonian obey the factorization formula

$$\begin{aligned}\langle \pi K | Q_i | B \rangle = {} & F_0^{B\to\pi}\, T^I_{K,i} \otimes f_K \Phi_K + F_0^{B\to K}\, T^I_{\pi,i} \otimes f_\pi \Phi_\pi + \\ & + T_i^{II} \otimes f_B \Phi_B \otimes f_K \Phi_K \otimes f_\pi \Phi_\pi \,, \end{aligned} \tag{1.1}$$

where Φ_M are leading-twist light-cone distribution amplitudes, and the $\otimes$-products imply integration over the light-cone momentum fractions of the constituent quarks inside the mesons. The hard-scattering kernels T_i are perturbatively calculable quantities. The factorization formula is true to all orders in perturbation theory, but it receives power corrections in $\Lambda_{\rm QCD}/m_b$. If QCD factorization is to be used as a tool for extracting the flavor parameters of the Standard Model, it is important to show that these power corrections are not anomalously large. Some sources of power corrections were identified and estimated in [2, 3, 4]. In [5, 6], the renormalon calculus was used to study power corrections due to soft gluon exchange in the decays $B \to D^{(*)}L$, where L is a light meson. Here we generalize and extend these analyses to focus on soft gluon corrections to the amplitudes for B decays into two light pseudoscalar mesons.

The model of power corrections that we develop relies on the analysis of infrared (IR) renormalon singularities, which arise because the low-momentum regions in loop diagrams give increasingly important contributions at higher orders in perturbation theory [7, 8, 9], rendering perturbation theory a divergent asymptotic expansion. IR renormalons are most transparently studied through singularities in the Borel transforms of perturbation series with respect to the coupling constant [10]. The Borel transform $S(u)$ of the perturbation series for a physical quantity has poles (or branch cuts) at half-integer values of the Borel variable u, which lead to ambiguities of order $(\Lambda_{\rm QCD}/Q)^{2u}$ when performing the inverse transform, a process to which we will refer as Borel resummation. (Here Q is the large momentum scale in the process.) Since the final answer for a physical quantity must be unambiguous, these renormalon ambiguities should be combined with power corrections of the same order in $1/Q$. Therefore, an analysis of IR renormalons may reveal non-trivial information about power corrections to various processes, which is particularly useful in cases where such corrections cannot be analyzed using an operator product expansion (see [11]–[16] for early applications of this idea).

We calculate the Borel transforms of the hard-scattering kernels appearing in the QCD factorization formula using the so-called "large-β_0 limit", in which the momentum flow through gluon lines is traced by inserting fermion bubbles on the propagators, replacing $n_f \to -3\beta_0/2$ at the end of the calculation [9]. In this way, the diagrams are effectively computed using a momentum-dependent running coupling at the vertices. This procedure gives the dominant higher-order contributions to a perturbation series in the very formal limit where $n_f \to -\infty$, in which case $\beta_0 \to \infty$ with fixed number of colors. Although for real QCD $\beta_0 = 11 - \frac{2}{3}n_f$ is not a large parameter, the use of the large-β_0 limit is motivated by the empirical observation that perturbation series are often dominated by terms of the form $\beta_0^{n-1}\alpha_s^n$.

In this paper, we calculate both the Wilson coefficients and the hard-scattering kernels entering the QCD factorization formula at next-to-leading order (NLO) in the large-β_0 limit. This is necessary for obtaining renormalization-group (RG) invariant expressions for the physical decay amplitudes. The expansion of the Wilson coefficients in the large-β_0 limit is a novel feature of our work, which goes beyond the approach taken in [5, 6]. The numerical study in section 4 helps to elucidate the main results of our work. We find that the power corrections to QCD factorization due to soft-gluon exchange are small, typically of order $(\Lambda_{\rm QCD}/m_b)^2$. The only exception is a first-order power correction to the hard-spectator term, which is to be expected considering that the effective scale for gluon exchange with the spectator quark is of order $\sqrt{\Lambda_{\rm QCD}m_b}$ [3]. Perhaps our most important result is that the strong-interaction phases of the decay amplitudes increase significantly when going beyond one-loop order in perturbation theory. This can potentially enhance the direct CP asymmetries in decays such as $B \to \pi\pi$ and $B \to \pi K$ with respect to the predictions found at NLO in RG-improved perturbation theory in [3]. Whether or not

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such increased asymmetries are realistic depends on whether there are significant multi-loop contributions not captured in the large-β_0 limit. We leave a more careful analysis of such contributions for future work.

2. Wilson coefficients in the large-β_0 limit

The QCD factorization formula provides model-independent expressions for hadronic weak decay amplitudes at leading power in the heavy-quark expansion. All non-trivial strong-interaction effects, which give corrections to the so-called "naive" factorization (vacuum insertion) approximation, are contained in RG-invariant parameters a_i, which are products of Wilson coefficients with convolutions of hard-scattering kernels and meson light-cone distribution amplitudes. In order to obtain RG-invariant results, it is crucial that the Wilson coefficients and hard-scattering kernels be treated in the same perturbative approximation scheme. Since our goal is to resum the perturbation series for the hard-scattering kernels in the large-β_0 limit, we must first compute the Wilson coefficients appearing in the effective weak hamiltonian in the same limit. This is non-trivial, since the counting scheme underlying the large-β_0 limit is different from the usual counting of logarithms used in the calculation of the Wilson coefficients. Specifically, whereas $\alpha_s \ln(M/\mu)$ is considered O(1) in the usual RG counting, this quantity is $\mathrm{O}(1/\beta_0)$ in the large-β_0 limit. On the contrary, whereas $1/\ln(M/\mu)$ is usually treated as a small parameter, it is considered O(1) in the large-β_0 limit.

Consider, as an example, the rare hadronic decays $B \to \pi\pi$, for which the effective weak hamiltonian (neglecting small contributions from electroweak penguins) is [17]

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \left(\sum_{i=1,2} C_i(\mu)\, Q_i^p + \sum_{i=3,\dots,6} C_i(\mu)\, Q_i + C_{8g}(\mu)\, Q_{8g} \right), \qquad (2.1)$$

where $\lambda_p = V_{pb} V_{pd}^*$ are products of CKM matrix elements. The scale dependence of the Wilson coefficients is governed by the evolution equation (in compact matrix notation)

$$\vec{C}(\mu) = T_\alpha \exp\left[- \int_{\alpha_s(M)}^{\alpha_s(\mu)} d\alpha\, \frac{\boldsymbol{\gamma}^T(\alpha)}{2\alpha\, \beta(\alpha)} \right] \vec{C}(M)\,, \qquad (2.2)$$

where the evolution matrix is given in terms of an ordered matrix exponential involving the anomalous dimension matrix $\boldsymbol{\gamma}$ and the β-function. The initial values $\vec{C}(M)$ at a scale $M \sim m_W$ are obtained by matching the Standard Model onto a low-energy effective theory, in which the heavy fields of the electroweak gauge bosons and the top quark are integrated out.

To evaluate (2.2) at subleading order in the large-β_0 limit, we expand the QCD β-function as $\beta(\alpha) = b + \mathrm{O}(1/\beta_0)$, where the rescaled coupling

$$b(\mu) = \beta_0\, \frac{\alpha_s(\mu)}{4\pi} = \frac{1}{\ln\left(\mu^2/\Lambda^2_{\overline{\text{MS}}}\right)} \qquad (2.3)$$

is O(1) in the large-β_0 limit. Next, we change variables from α to b in the integral in (2.2), write the anomalous dimension matrix as a function of b, and expand in powers of $1/\beta_0$. Taking into account that factors of n_f must be replaced with $-3\beta_0/2$, we find that in 2×5 block form the resulting matrix is given by

$$\boldsymbol{\gamma}^T(b) = \left(\begin{array}{c|c} 0 & 0 \\ \hline 0 & b\,\boldsymbol{C_0} \end{array}\right) + \frac{1}{\beta_0}\left(\begin{array}{c|c} \boldsymbol{A_1}(b) & 0 \\ \hline b\,\boldsymbol{B_1} & \boldsymbol{C_1}(b) \end{array}\right) + \mathrm{O}\left(\frac{1}{\beta_0^2}\right). \tag{2.4}$$

The justification for writing the leading term in this expansion as a constant 5×5 matrix $\boldsymbol{C_0}$ times the coupling constant b is that this corner of the anomalous dimension matrix does not pick up an additional factor of n_f when going from leading order (LO) to next-to-leading order in perturbation theory, so that all further perturbative contributions are absorbed in the matrix $\boldsymbol{C_1}(b)$, the form of which is irrelevant to our discussion. A similar argument applies to the submatrix in the lower left corner, which at order $1/\beta_0$ is given by a constant 5×2 matrix $\boldsymbol{B_1}$ times b.

The matrix $\boldsymbol{A_1}(b)$ governing the evolution of the current-current operators $Q_{1,2}^p$ in the large-β_0 limit is a non-trivial function of the coupling, which has been calculated in [6, 18]. The result is

$$\boldsymbol{A_1}(b) = \gamma(b)\begin{pmatrix} -\frac{1}{N} & 1 \\ 1 & -\frac{1}{N} \end{pmatrix}, \tag{2.5}$$

where $N=3$ is the number of colors, and

$$\gamma(b) = \frac{2b(3+2b)(1-b)\,\Gamma(4+2b)}{3\Gamma(1-b)\,\Gamma^2(2+b)\,\Gamma(3+b)}. \tag{2.6}$$

The matrices $\boldsymbol{C_0}$ and $\boldsymbol{B_1}$ can be derived from the expressions for the LO anomalous dimension matrices given in [17]:

$$\boldsymbol{C_0} = \begin{pmatrix} 0 & \frac{1}{N} & 0 & \frac{1}{N} & 0 \\ 0 & -1 & 0 & -1 & 0 \\ 0 & \frac{1}{N} & 0 & \frac{1}{N} & 0 \\ 0 & -1 & 0 & -1 & 0 \\ -\frac{9}{2} & -\frac{11N}{6}+\frac{29}{6N} & \frac{9}{2} & \frac{8N}{3}-\frac{25}{6N} & 0 \end{pmatrix}, \qquad \boldsymbol{B_1} = \begin{pmatrix} -\frac{2}{3N} & 0 \\ \frac{2}{3} & 0 \\ -\frac{2}{3N} & 0 \\ \frac{2}{3} & 0 \\ \frac{11N}{9}-\frac{29}{9N} & 3 \end{pmatrix}. \tag{2.7}$$

Taking into account the textures of these matrices as well as corresponding relations between the matching conditions for the various coefficients at the scale $\mu = m_W$, we find that the solutions for the Wilson coefficients can be expanded as

$$\begin{aligned} C_1(\mu) &= 1 + \frac{1}{\beta_0}\,C_1^{(1)}(\mu) + \mathrm{O}\left(\frac{1}{\beta_0^2}\right), \\ C_i(\mu) &= \frac{1}{\beta_0}\,C_i^{(1)}(\mu) + \mathrm{O}\left(\frac{1}{\beta_0^2}\right); \qquad i = 2,\dots,6, \\ C_{8g}(\mu) &= C_{8g}^{(0)}(\mu) + \frac{1}{\beta_0}\,C_{8g}^{(1)}(\mu) + \mathrm{O}\left(\frac{1}{\beta_0^2}\right), \end{aligned} \tag{2.8}$$

where

$$C_2^{(1)}(\mu) = -N\,C_1^{(1)}(\mu), \qquad C_4^{(1)}(\mu) = C_6^{(1)}(\mu) = -N\,C_3^{(1)}(\mu) = -N\,C_5^{(1)}(\mu). \tag{2.9}$$

Up to $\mathrm{O}(1/\beta_0)$, the Wilson coefficients $C_1,\dots,C_6$ are thus determined in terms of two

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NLO	C_1	C_2	C_3	C_4	C_5	C_6
$\mu = m_b/2$	1.137	-0.295	0.021	-0.051	0.010	-0.065
$\mu = m_b$	1.081	-0.190	0.014	-0.036	0.009	-0.042
$\mu = 2m_b$	1.045	-0.113	0.009	-0.025	0.007	-0.027
Large-β_0 limit	C_1	C_2	C_3	C_4	C_5	C_6
$\mu = m_b/2$	1.087	-0.260	0.020	-0.061	0.020	-0.061
$\mu = m_b$	1.061	-0.183	0.014	-0.041	0.014	-0.041
$\mu = 2m_b$	1.039	-0.118	0.009	-0.028	0.009	-0.028

Table 1: Comparison of the Wilson coefficients in the large-β_0 limit with those obtained at NLO in conventional, RG-improved perturbation theory. Input parameters are $m_t(m_t) = 167\,\text{GeV}$, $m_b(m_b) = 4.2\,\text{GeV}$, $\alpha_s(m_b) = 0.2244$, and $\alpha_s(m_W) = 0.1202$.

functions, given by

$$\begin{aligned} C_2^{(1)}(\mu) &= C_2^{(1)}(M) - \int_{b(M)}^{b(\mu)} db\, \frac{\gamma(b)}{2b^2}\,, \\ C_4^{(1)}(\mu) &= C_4^{(1)}(M) + \left(\frac{b(\mu)}{b(M)} - 1\right)\left(C_4^{(1)}(M) - \frac{1}{3}\right). \end{aligned} \tag{2.10}$$

The coefficient of the chromo-magnetic operator Q_{8g} will be needed only at LO, where it is scale independent, i.e.

$$C_{8g}^{(0)}(\mu) = C_{8g}^{(0)}(M)\,. \tag{2.11}$$

Note that, as a consequence of the fact that the matrix $\boldsymbol{C_0}$ has eigenvalues 0 and -2, the scale dependence of the penguin coefficients $C_3, \dots, C_6$ governed by the function $C_4^{(1)}(\mu)$ is extremely simple, and given by $b(\mu)/b(M) - 1 = 2b(\mu)\ln(M/\mu)$. We will see below that the appearance of a logarithm is indeed required to cancel the scale dependence of the hard-scattering kernels associated with penguin contractions of the operators Q_i.

The remaining step in the calculation of the Wilson coefficients is to specify the matching conditions at a scale $M \sim m_W$. In general, these will be complicated functions of the coupling $b(m_W)$, which is formally O(1) in the large-β_0 limit. However, since $b(m_W) \approx 0.07$ is numerically small, it will be a good approximation to keep only the one-loop contributions to the matching conditions. They read (in the NDR scheme)

$$C_2^{(1)}(m_W) \approx \frac{11}{2} b(m_W)\,, \qquad C_4^{(1)}(m_W) \approx \frac{1}{2}\widetilde{E}_0(x_t) b(m_W)\,, \qquad C_{8g}^{(0)}(m_W) \approx F(x_t)\,, \tag{2.12}$$

where $x_t = (m_t/m_W)^2$, and

$$\begin{aligned} \widetilde{E}_0(x) &= -\frac{2}{3}(1 + \ln x) + \frac{x(18 - 11x - x^2)}{12(1-x)^3} + \frac{x^2(15 - 16x + 4x^2)}{6(1-x)^4}\ln x\,, \\ F(x) &= -\frac{x(2 + 5x - x^2)}{8(1-x)^3} - \frac{3x^2}{4(1-x)^4}\ln x\,. \end{aligned} \tag{2.13}$$

In table 1, we compare our results for the Wilson coefficients obtained at subleading order

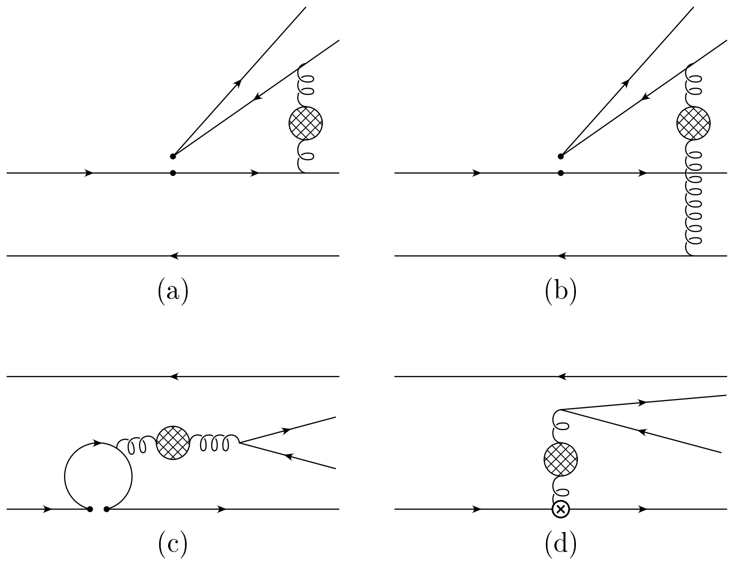

Figure 1: Representative examples of contributions to the hard-scattering kernels in the QCD factorization formula including bubble resummation of the gluon propagators: (a) vertex contribution, (b) hard-scattering with the spectator quark, (c) penguin contribution, (d) chromo-magnetic dipole contribution.

in the large-β_0 limit with those derived using the standard RG evolution at NLO. Given the great simplifications that occur in the large-β_0 limit, it is remarkable that the numerical values obtained in the two schemes are very close to each other.

3. Borel resummation for the decay amplitudes

Having obtained expressions for the Wilson coefficients in the large-β_0 limit, our next task is to calculate the hard-scattering kernels in the QCD factorization formula using the same resummation scheme. There are three different types of kernels, corresponding to "non-factorizable" vertex corrections, "non-factorizable" gluon exchange with the spectator quark in the B meson, and penguin contributions. Representative examples of each kind are depicted in figure 1, where (c) and (d) are both part of the penguin contribution. The weak decay amplitudes are obtained by evaluating the hadronic matrix elements of a "factorized transition operator" consisting of products of the parameters a_i and two quark currents, whose matrix elements are known in terms of meson decay constants and semi-leptonic form factors [2, 3]. As an example, we quote the resulting expressions for the $B \to \pi\pi$ decay amplitudes. They are

$$\begin{aligned} -\mathcal{A}(\bar{B}^0 \to \pi^+\pi^-) &= \left[\lambda_u\, a_1 + \sum_{p=u,c} \lambda_p\, (a_4^p + r_\chi\, a_6^p) \right] A_{\pi\pi} \,, \\ -\sqrt{2}\,\mathcal{A}(B^- \to \pi^-\pi^0) &= \lambda_u\, (a_1 + a_2)\, A_{\pi\pi} \,, \\ \mathcal{A}(\bar{B}^0 \to \pi^0\pi^0) &= \sqrt{2}\,\mathcal{A}(B^- \to \pi^-\pi^0) - \mathcal{A}(\bar{B}^0 \to \pi^+\pi^-) \,, \end{aligned} \tag{3.1}$$

where

$$A_{\pi\pi} = i\frac{G_F}{\sqrt{2}}\,(m_B^2 - m_\pi^2)\,F_0^{B\to\pi}(m_\pi^2)\,f_\pi\,, \qquad r_\chi = \frac{2m_\pi^2}{m_b(m_u+m_d)}\,, \tag{3.2}$$

and we have neglected small electroweak penguin contributions and power-suppressed weak annihilation terms. Similar results hold for other decays such as $B\to\pi K$. The general expressions for the a_i parameters at leading order in the heavy-quark expansion read (we quote results for $B\to\pi K$ decays; those for $B\to\pi\pi$ are obtained by replacing $K\to\pi$ everywhere)

$$\begin{aligned}
a_1 &= C_1(\mu) + \frac{C_2(\mu)}{N}\left[1 + \frac{C_F\alpha_s(\mu)}{4\pi}\,V_K(\mu) + \frac{C_F\pi\alpha_s(\mu)}{N}\,H_{K\pi}(\mu)\right],\\
a_2 &= C_2(\mu) + \frac{C_1(\mu)}{N}\left[1 + \frac{C_F\alpha_s(\mu)}{4\pi}\,V_\pi(\mu) + \frac{C_F\pi\alpha_s(\mu)}{N}\,H_{\pi K}(\mu)\right],\\
a_4^p &= C_4(\mu) + \frac{C_3(\mu)}{N}\left[1 + \frac{C_F\alpha_s(\mu)}{4\pi}\,V_K(\mu) + \frac{C_F\pi\alpha_s(\mu)}{N}\,H_{K\pi}(\mu)\right] + \frac{C_F\alpha_s(\mu)}{4\pi}\,\frac{P_{K,2}^p(\mu)}{N}\,,\\
a_6^p &= C_6(\mu) + \frac{C_5(\mu)}{N}\left[1 - 6\cdot\frac{C_F\alpha_s(\mu)}{4\pi}\right] + \frac{C_F\alpha_s(\mu)}{4\pi}\,\frac{P_{K,3}^p(\mu)}{N}\,.
\end{aligned} \tag{3.3}$$

The quantities V, H and P include the NLO corrections from the vertex, hard-spectator and penguin diagrams, respectively [3]. The subscript $n=2,3$ on $P_{K,n}^p$ refers to the twist of the corresponding light-cone distribution amplitude of the kaon.

3.1 General results for the Borel transforms

The summation of terms of order $\beta_0^{n-1}\alpha_s^n$ in the kernels V, H and P is achieved by repeating the calculations of [3] inserting an arbitrary number of fermion loops on the gluon propagators (see [6] for details). The results are then written as integrals over the Borel variable u. Using the fact that $\alpha_s = \mathrm{O}(1/\beta_0)$ and the explicit results for the Wilson coefficients obtained in the previous section, it follows that

$$a_1 = 1 + \mathrm{O}\left(\frac{1}{\beta_0^2}\right) \tag{3.4}$$

becomes trivial in the large-β_0 limit. Also, the parameters a_3 and a_5, which do not enter the $B\to\pi\pi$ decay amplitudes in (3.1), vanish to this order, i.e. a_3, $a_5 = \mathrm{O}(1/\beta_0^2)$. These results compare well with the full NLO analysis in [3], where it was found that $a_1 \approx 1.03 + 0.02i$, while the values of a_3 and a_5 are below 1% in magnitude.

The results for the three remaining coefficients are non-trivial. In the case of a_2, both the vertex term V_π and the hard-spectator term $H_{\pi K}$ contribute. The vertex term has been computed in the large-β_0 limit in [6], with the result that

$$\begin{aligned}
\frac{\alpha_s(\mu)}{4\pi}\,V_\pi(\mu) \longrightarrow \frac{2}{\beta_0}\Bigg[&\int_0^{b(\mu)}\frac{db}{b}\left(\frac{\gamma(b)}{2b} - 3\right) + 3\ln\frac{b(\mu)}{b(m_b)} +\\
&+ \int_0^\infty du\, e^{-u/b(m_b)}\, e^{5u/3}\, S_V(u)\Bigg]\,,
\end{aligned} \tag{3.5}$$

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and the Borel transform

$$S_V(u) = \int_0^1 dx\,\Phi_\pi(x)\left[\frac{3}{u}\,e^{-5u/3} - \frac{(3-u)\,x^{-u}}{(1-u)(2-u)}(\pi\cot\pi u + i\pi) - \right.$$
$$\left. - \frac{(3-u)\Gamma(2-2u)\Gamma(u)}{\Gamma(3-u)}\int_0^1 dz\,\frac{1}{[z(x+(1-x)z)]^u}\right]. \qquad (3.6)$$

Here and below we take advantage of the symmetry of the pion distribution amplitude under $x \leftrightarrow (1-x)$ to simplify the results. The term proportional to $e^{-5u/3}$ subtracts the pole at $u=0$, as appropriate in the $\overline{\rm MS}$ scheme. In the large-β_0 limit, the hard-spectator contribution is obtained from bubble resummation on the gluon propagator of a tree diagram, so the result is scale independent and given by

$$\frac{\pi\alpha_s(\mu)}{N}\,H_{\pi K}(\mu) \longrightarrow \frac{2}{\beta_0}\int_0^\infty du\,e^{-u/b(m_b)}\,e^{5u/3}\,S_H(u)\,, \qquad (3.7)$$

where (we use the short-hand notation $\bar y \equiv 1-y$)

$$S_H(u) = \frac{2\pi^2}{N}\,\frac{f_B f_K}{m_B^2\,F_0^{B\to K}(0)}\int_0^1 dx\int_0^1 dy\int_0^1 d\xi\,\frac{\Phi_\pi(x)\,\Phi_K(y)\,\Phi_B(\xi)}{x\,(\bar y\,\xi)^{1+u}}\,. \qquad (3.8)$$

Combining the various pieces, it is readily seen that the scale dependence cancels between the Wilson coefficients and the kernels. The RG-invariant result is

$$a_2 = \frac{1}{N} + \frac{2C_F}{N\beta_0}\left\{C_2^{(1)}(m_b) + \int_0^{b(m_b)}\frac{db}{b}\left(\frac{\gamma(b)}{2b} - 3\right) + \right.$$
$$\left. + \int_0^\infty du\,e^{-u/b(m_b)}\,e^{5u/3}\,[S_V(u) + S_H(u)]\right\} + \mathrm{O}\left(\frac{1}{\beta_0^2}\right). \qquad (3.9)$$

Let us now discuss the resummation for the penguin coefficients $a_{4,6}^p$. It follows from our earlier results that in this case the vertex and hard-spectator contributions enter only at $\mathrm{O}(1/\beta_0^2)$, and so it suffices to calculate the penguin kernels $P_{K,n}^p$ in (3.3). The cancellation of the scale dependence between the Wilson coefficients and these kernels is subtle and relies in an essential way on our treatment of the Wilson coefficients in the large-β_0 limit. Consider the coefficient a_4^p as an example. For fixed momentum fraction y, we find[1]

$$\beta_0\,a_4^p(y) = C_4^{(1)}(\mu) + \frac{C_3^{(1)}(\mu)}{N} + \frac{C_F}{N}\int_0^\infty du\,e^{-u/b(m_b)}\,e^{5u/3}\,(-\bar y - i\epsilon)^{-u}\,\times$$
$$\times\left\{\frac{n_f}{\beta_0}\left[C_4^{(1)}(\mu) + C_6^{(1)}(\mu)\right]\left(\frac{4}{3}\ln\frac{m_b}{\mu} - G(0,\bar y)\right) + \right.$$
$$\left. + C_1^{(0)}\left[\frac{4}{3}\ln\frac{m_b}{\mu} + \frac{2}{3} - G(s_p,\bar y)\right] - \frac{2C_{8g}^{(0)}}{\bar y}\right\} + \mathrm{O}\left(\frac{1}{\beta_0}\right), \qquad (3.10)$$

[1]For the sake of this perturbative argument it is legitimate to interchange the order of the integrations over u and y.

where $s_p = (m_p/m_b)^2$ for $p = u, c$, and

$$G(s, \bar{y}) = -4 \int_0^1 dz\, z(1-z) \ln[s - z(1-z)\bar{y} - i\epsilon]\,. \tag{3.11}$$

Noting that $G(0, \bar{y}) = \frac{2}{3}[\frac{5}{3} - \ln(-\bar{y} - i\epsilon)]$, replacing $n_f/\beta_0 \to -3/2$, performing the integral over u, and using the results from section 2, we obtain

$$\begin{aligned}
\beta_0\, a_4^p(y) &= \frac{2C_F}{N} C_4^{(1)}(\mu) + \frac{2C_F}{N}\, b(-\bar{y}\, e^{-5/3}\, m_b^2 - i\epsilon) \times \\
&\quad \times \left\{ C_4^{(1)}(\mu) \left(-2\ln\frac{m_b}{\mu} + \frac{5}{3} - \ln(-\bar{y} - i\epsilon) \right) + \right. \\
&\qquad \left. + \frac{2}{3}\ln\frac{m_b}{\mu} + \frac{1}{3} - \frac{1}{2}\, G(s_p, \bar{y}) - \frac{C_{8g}^{(0)}}{\bar{y}} \right\} + \mathrm{O}\left(\frac{1}{\beta_0}\right) \qquad (3.12) \\
&= \frac{2C_F}{N} \left\{ C_4^{(1)}(M) + b(M) \left[\frac{1}{3} + \frac{1}{2}\, G(0, \bar{y}) - \frac{1}{2}\, G(s_p, \bar{y}) - \frac{C_{8g}^{(0)}}{\bar{y}} \right] \right\} + \mathrm{O}\left(\frac{1}{\beta_0}\right),
\end{aligned}$$

where $M^2 = -\bar{y}\, e^{-5/3}\, m_b^2 - i\epsilon$, and in the last step we have used the evolution equation for $C_4^{(1)}$ in (2.10). It follows that $a_4^p(y)$ is indeed independent of the scale μ. The terms proportional to $C_4^{(1)}(\mu) + C_6^{(1)}(\mu)$ in (3.10) are absorbed by setting the scale in the Wilson coefficients to be the (time-like) momentum M carried by the gluon propagator. Having established that the resummed expression for a_4^p is scale independent, we return to the original expression in (3.10), set $\mu = m_b$, and integrate over the light-cone variable y to obtain the final results

$$\begin{aligned}
a_4^p &= \frac{2C_F}{N\beta_0} \left[C_4^{(1)}(m_b) + \int_0^\infty du\, e^{-u/b(m_b)}\, e^{5u/3}\, S_{P,2}^p(u) \right] + \mathrm{O}\left(\frac{1}{\beta_0^2}\right), \\
a_6^p &= \frac{2C_F}{N\beta_0} \left[C_4^{(1)}(m_b) + \int_0^\infty du\, e^{-u/b(m_b)}\, e^{5u/3}\, S_{P,3}^p(u) \right] + \mathrm{O}\left(\frac{1}{\beta_0^2}\right),
\end{aligned} \tag{3.13}$$

where

$$\begin{aligned}
S_{P,2}^p(u) &= \int_0^1 dy\, \Phi_K(y)\, \frac{e^{i\pi u}}{\bar{y}^u} \left[\frac{1}{3} + \frac{3}{2}\, C_4^{(1)}(m_b)\, G(0, \bar{y}) - \frac{1}{2}\, G(s_p, \bar{y}) - \frac{C_{8g}^{(0)}}{\bar{y}} \right], \\
S_{P,3}^p(u) &= \int_0^1 dy\, \Phi_K^p(y)\, \frac{e^{i\pi u}}{\bar{y}^u} \left[\frac{1}{3} + \frac{3}{2}\, C_4^{(1)}(m_b)\, G(0, \bar{y}) - \frac{1}{2}\, G(s_p, \bar{y}) - C_{8g}^{(0)} \right].
\end{aligned} \tag{3.14}$$

Equations (3.4), (3.9) and (3.13) are the main results of this work. They allow us to study the effects of higher-order $\beta_0^{n-1}\alpha_s^n$ corrections on the weak decay amplitudes for processes such as $B \to \pi\pi$ and $B \to \pi K$. Compared with the general expressions in (3.3), the terms that survive at NLO in the large-β_0 limit are rather simple. However, these terms still capture the bulk of the non-trivial strong-interaction effects in non-leptonic B decays. In particular, it has already been noted in [2, 3] that a_1 is always very close to 1, that hard-spectator interactions are the most important for the coefficient a_2, and that the penguin matrix elements give the dominant NLO contributions to $a_{4,6}^p$. Before we evaluate our results numerically, we now simplify the expressions for the Borel transforms by using a model for the meson distribution amplitudes.

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3.2 Results with asymptotic distribution amplitudes

The exact integral expressions for the Borel transforms of the hard-scattering kernels simplify if the asymptotic forms of the pion and kaon light-cone distribution amplitudes are adopted. They are

$$\Phi_\pi(x) = 6x(1-x)\,, \qquad \Phi_K(y) = 6y(1-y)\,, \qquad \Phi_K^p(y) = 1\,. \tag{3.15}$$

In addition, for the hard-spectator term we employ a simple model for the leading-twist distribution amplitude of the B-meson suggested in [19], according to which

$$\Phi_B(\xi) = \frac{\xi}{\xi_0^2}\, e^{-\xi/\xi_0}\,; \qquad \xi_0 = \frac{\lambda_B}{m_B}\,. \tag{3.16}$$

The hadronic parameter $\lambda_B = \mathrm{O}(\Lambda_{\mathrm{QCD}})$ is defined in terms of the first inverse moment, $\int_0^1 (d\xi/\xi)\,\Phi_B(\xi) = m_B/\lambda_B$. Its value at present is rather uncertain, a typical range being $\lambda = (350 \pm 150)\,\mathrm{GeV}$.

The results obtained for the Borel transforms of the vertex and hard-spectator terms after integration over these distribution amplitudes are

$$\begin{aligned}
S_V(u) &= \frac{3}{u}\, e^{-5u/3} - \frac{6\pi(\cot \pi u + i)}{(1-u)(2-u)^2} - \\
&\quad - \frac{6\Gamma(1-2u)\,\Gamma(u)}{(2-u)\,\Gamma(3-u)} \left[1 + 2u(1-2u) \left(\psi(1-2u) - \psi(1-u) - \frac{3}{4} \right) \right], \\
S_H(u) &= \frac{18\pi^2}{N}\, \frac{f_B f_K}{m_B \lambda_B\, F_0^{B\to K}(0)}\, \frac{2\Gamma(1-u)}{(1-u)(2-u)} \left(\frac{m_B}{\lambda_B} \right)^u ,
\end{aligned} \tag{3.17}$$

where $\psi(z)$ is the digamma function, defined as the logarithmic derivative of the gamma function. The results for the penguin contributions read

$$\begin{aligned}
S_{P,2}^p(u) &= e^{i\pi u} \left[\frac{2}{(2-u)(3-u)} + \frac{3}{2}\, C_4^{(1)}(m_b)\, \mathcal{G}_K(0,u) - \frac{1}{2}\, \mathcal{G}_K(s_p,u) - \frac{6C_{8g}^{(0)}}{(1-u)(2-u)} \right], \\
S_{P,3}^p(u) &= e^{i\pi u} \left[\frac{1}{3(1-u)} + \frac{3}{2}\, C_4^{(1)}(m_b)\, \widehat{\mathcal{G}}_K(0,u) - \frac{1}{2}\, \widehat{\mathcal{G}}_K(s_p,u) - \frac{C_{8g}^{(0)}}{1-u} \right],
\end{aligned} \tag{3.18}$$

where we have defined

$$\mathcal{G}_K(s,u) = \int_0^1 dy\, \Phi_K(y)\, \bar{y}^{-u}\, G(s,\bar{y})\,, \qquad \widehat{\mathcal{G}}_K(s,u) = \int_0^1 dy\, \Phi_K^p(y)\, \bar{y}^{-u}\, G(s,\bar{y})\,. \tag{3.19}$$

For $p = u$, these functions can be evaluated in closed form, yielding

$$\begin{aligned}
\mathcal{G}_K(0,u) &= \frac{4}{(2-u)(3-u)} \left(\frac{5}{3} + i\pi + \frac{1}{2-u} + \frac{1}{3-u} \right), \\
\widehat{\mathcal{G}}_K(0,u) &= \frac{2}{3(1-u)} \left(\frac{5}{3} + i\pi + \frac{1}{1-u} \right).
\end{aligned} \tag{3.20}$$

For the case of the charm-quark loop, the functions $\mathcal{G}_K(s_c,u)$ and $\widehat{\mathcal{G}}_K(s_c,u)$ with $s_c \neq 0$ do not have a simple analytic form. We choose instead to write them as infinite sums of poles in the Borel plane, by expanding the logarithm in (3.11) in powers of $\bar{y}$ and then performing the integrals. We find that

$$\begin{aligned}
\mathcal{G}_K(s_c,u) &= \frac{-4\ln s_c}{(2-u)(3-u)} + \sum_{n=1}^{\infty} \frac{\sqrt{\pi}}{2n}\,(4s_c)^{-n}\,\frac{\Gamma(n+2)}{\Gamma(n+\frac{5}{2})}\,\frac{6}{(2+n-u)(3+n-u)}\,, \\
\widehat{\mathcal{G}}_K(s_c,u) &= \frac{-2\ln s_c}{3(1-u)} + \sum_{n=1}^{\infty} \frac{\sqrt{\pi}}{2n}\,(4s_c)^{-n}\,\frac{\Gamma(n+2)}{\Gamma(n+\frac{5}{2})}\,\frac{1}{1+n-u}\,.
\end{aligned} \tag{3.21}$$

The sums converge as long as $s_c > 1/4$ and can be expressed in terms of hypergeometric functions. These functions can then be analytically continued to the physical region $s_c < 1/4$, where they develop a non-zero imaginary part (recall that $s_c \to s_c - i\epsilon$).

3.3 IR renormalon ambiguities and power corrections

Expansion of the Borel transforms (times $e^{5u/3}$) in powers of u generates the perturbative series of the kernels in the $\overline{\text{MS}}$ scheme. The Borel sums of these series are obtained by performing the u-integrals exactly. However, the Borel transforms contain poles along the integration contour, which render the results of Borel summation ambiguous. This is the well-known problem of IR renormalons, which reflects the fact that the original perturbation series were asymptotic.

For a given Borel transform, the residue of the nearest pole provides a measure of the renormalon ambiguity, which is conventionally taken as an estimate of the power corrections that should be added to the perturbation series in order to get a well-defined result. We will now work out the structure of these leading singularities for the parameters a_i. Note that the product

$$e^{-u/b(m_b)}\,e^{5u/3} = \left(\frac{e^{5/6}\Lambda_{\overline{\text{MS}}}}{m_b}\right)^{2u} \equiv \left(\frac{\Lambda_V}{m_b}\right)^{2u} \tag{3.22}$$

is RG invariant. The constant $5/6$ arises in the calculation of the fermion-loop contribution to the gluon self-energy in the $\overline{\text{MS}}$ scheme and compensates the scheme dependence of $\Lambda_{\overline{\text{MS}}}$. We will thus express our results in terms of the invariant scale parameter Λ_V.

If the light-cone distribution amplitudes are given by the model used in the explicit calculations of section 3.2, then all of the Borel transforms have a leading pole at $u = 1$. For the hard-spectator term this leads to a power correction of order Λ_V/m_b, which is expected, because the effective scale in the hard-spectator terms is not m_b^2 but $\mu_h^2 \simeq \Lambda_{\text{QCD}} m_b$ [3]. In all other cases the leading power correction is of order $(\Lambda_V/m_b)^2$. Expanding the Borel transforms about the positions of the poles, and including terms up to second order in the heavy-quark expansion, we find

$$\begin{aligned}
\Delta a_2 &= \frac{12C_F}{N\beta_0}\,\frac{\Lambda_V^2}{m_b^2}\left(\ln\frac{m_b}{\Lambda_V} - \frac{1}{4} - i\pi\right) + \frac{12C_F}{N\beta_0}\,\frac{6\pi^2}{N}\,\frac{f_B f_K}{m_B\lambda_B\,F_0^{B\to K}(0)} \times \\
&\quad \times \left\{\frac{\Lambda_H}{m_b}\left(\ln\frac{m_b}{\Lambda_H} - \gamma_E - 1\right) + \frac{\Lambda_H^2}{m_b^2}\left(\ln\frac{m_b}{\Lambda_H} - \gamma_E + 2\right)\right\} + \cdots\,, \\
\Delta a_4^p &= \frac{12C_F}{N\beta_0}\,\frac{\Lambda_V^2}{m_b^2}\,C_{8g}^{(0)} + \cdots\,,
\end{aligned}$$

$$\Delta a_6^c = \frac{2C_f}{N\beta_0}\frac{\Lambda_V^2}{m_b^2}\left\{C_{8g}^{(0)} + \frac{1}{3}\left(2\ln\frac{m_b}{m_c} - 1\right) - \left(2\ln\frac{m_b}{\Lambda_V} + \frac{5}{3}\right)C_4^{(1)}(m_b)\right\} + \cdots\,,$$
$$\Delta a_6^u = \frac{2C_f}{N\beta_0}\frac{\Lambda_V^2}{m_b^2}\left\{C_{8g}^{(0)} + \left(2\ln\frac{m_b}{\Lambda_V} + \frac{5}{3}\right)\left[\frac{1}{3} - C_4^{(1)}(m_b)\right] - \frac{1}{3}\right\} + \cdots\,, \qquad (3.23)$$

where $\Lambda_H = (m_B/m_b)(\Lambda_V^2/\lambda_B) = \mathrm{O}(\Lambda_{\mathrm{QCD}})$ in the hard-spectator contribution to Δa_2. These expressions may serve as a model for the leading power corrections to the decay amplitudes arising from "non-factorizable" soft gluon exchange. Because most of the corrections are suppressed by two powers of Λ_V/m_b, their numerical effects turn out to be rather small. Only a_2 receives a first-order correction that can be sizeable. However, the effect of that correction can be absorbed into a redefinition of the hadronic parameter λ_B, which in any case is poorly determined theoretically. Therefore, power corrections from soft gluon exchange have a minor impact on the phenomenological analysis of charmless hadronic B decays in the framework of QCD factorization.

Let us briefly discuss the changes that occur if the light-cone distribution amplitudes are allowed to have an arbitrary endpoint behavior, such as

$$\Phi_M(x) \simeq \bar{x}^{\delta_M}\,[1 + \mathrm{O}(\bar{x})] \qquad (3.24)$$

as $x \to 1$ (and similarly for $x \to 0$). The model used above corresponds to $\delta_\pi, \delta_K, \delta_B = 1$ and $\delta_K^p = 0$. These results would continue to hold at any fixed order in a Gegenbauer expansion of the distribution amplitudes, but may be altered if an infinite number of Gegenbauer moments is included. The effect of a more general endpoint behavior on the position of the poles in the Borel plane depends on the function being considered. As seen from (3.6), the Borel transform corresponding to the vertex function V_M has a pole at $u = 1$ regardless of the endpoint behavior of the distribution amplitude of the meson M. For the penguin and hard-spectator terms, on the other hand, changing the distribution amplitudes will change the position of the leading IR renormalon poles. For the penguin terms the modifications are simple: the leading poles in the twist-2 and twist-3 penguin contributions are shifted from $u = 1$ to $u = \delta_K$ (twist-2) and $u = 1 + \delta_K^p$ (twist-3), respectively. The effects on the hard-scattering term are slightly more involved. If the B-meson and kaon distribution amplitudes have the same endpoint behavior, then there are both single and double poles at $u = \delta_K = \delta_B$. On the other hand, if δ_B differs from δ_K, then the position of the leading simple pole is determined by whichever is smaller.

3.4 Fermion loops beyond the large-β_0 approximation

Whereas in most applications of the large-β_0 limit the replacement $n_f \to -3\beta_0/2$ after the calculation of fermion bubble graphs can be motivated by a diagrammatic analysis, this replacement is hard to justify for the penguin coefficients entering the analysis of non-leptonic B decays. The fermion-loop contributions proportional to n_f are usually accompanied by gauge boson and ghost loops. Calculations using bubble resummation are often good approximations to full QCD because the replacement $n_f \to -3\beta_0/2$ (approximately) accounts for the extra diagrams not related to fermion bubbles. The factor of n_f associated with the penguin coefficients is not of this nature. It arises from the contraction of two fermion fields of a 4-quark operator in the effective weak hamiltonian. Such fermion loops are not accompanied in an obvious way by other QCD loop diagrams.[2] To produce reasonable numerical results for the Borel-resummed kernels will require moving beyond the large-β_0 approximation.

A proper counting of fermionic penguin contractions can be achieved *a posteriori* by making the replacement

$$\frac{3}{2}\,C_4^{(1)}(m_b)\,G(0,\bar{y}) \longrightarrow -\frac{C_4^{(1)}(m_b)}{\beta_0}\,\left[(n_f-2)\,G(0,\bar{y})+G(s_c,\bar{y})+G(1,\bar{y})\right] \qquad (3.25)$$

in the expressions for the Borel transforms $S^p_{P,2}$ and $S^p_{P,3}$ in (3.14). Upon setting $n_f = 5$, (3.25) accounts for loop contributions from three massless quarks, the charm quark, and the bottom quark (cf. [3, eqs. (49) and (54)]). Formally, the two sides of (3.25) coincide in the large-β_0 limit, and so making this replacement is equivalent to adding some higher-order terms to the renormalon calculus. Since there are other terms of subleading order in the large-β_0 limit not taken into account, this procedure may seem arbitrary. Nevertheless, the above replacement is physically motivated and, in particular, reproduces the proper imaginary parts and heavy-quark mass dependence of the fermion loops. From now on, we will refer to the scheme defined by (3.25) as the "modified" large-β_0 limit.

A replacement analogous to (3.25) must be made in (3.18). The resulting expressions for the leading IR renormalon ambiguities in the coefficients a_6^p also change. In the last two equations in (3.23), we must replace

$$-\left(2\ln\frac{m_b}{\Lambda_V}+\frac{5}{3}\right)C_4^{(1)}(m_b) \longrightarrow \frac{2C_4^{(1)}(m_b)}{3\beta_0}\left[(n_f-2)\left(2\ln\frac{m_b}{\Lambda_V}+\frac{5}{3}\right)+2\ln\frac{m_b}{m_c}\right]. \qquad (3.26)$$

4. Numerical results

In addition to providing a model for power corrections, the results of the previous sections allow us to study the numerical significance of higher-order perturbative contributions to the hard-scattering kernels in the large-β_0 limit, which may be considered as a toy model for the asymptotic behavior of perturbation theory. Partial perturbation series for the quantities a_i, including terms up to order $\beta_0^{n-1}\alpha_s^n$, are obtained by retaining only terms up to $\mathrm{O}(u^{n-1})$ in the Taylor expansions for the Borel transforms about $u = 0$. The resummed series (in the large-β_0 limit) are obtained by performing the Borel integrals exactly, regularizing the singularities along the integration contour using the principal value prescription. The sum of the residues of the poles at half-integer values of u provides an estimate of the irreducible ambiguity in the definition of the resummed perturbation series. Whereas only the leading poles were considered in the previous section, exact expressions for the sum of all residues are used in our numerical analysis.

In table 2, we show different perturbative approximations to the coefficients a_2 and $(a_4^p + r_\chi\, a_6^p)$ entering the $B \to \pi\pi$, πK decay amplitudes. The first row gives the results obtained using the standard QCD factorization approach described in [3]. The following

[2]Such diagrams would appear at two-loop order when one calculates matrix elements of the chromo-magnetic dipole operator. We have performed a partial analysis of such higher-order contributions and found their effects to be strongly suppressed.

	a_2	$a_4^u + r_\chi\, a_6^u$	$a_4^c + r_\chi\, a_6^c$
NLO	$0.086 - 0.084i$	$-0.081 - 0.034i$	$-0.091 - 0.014i$
1-loop	$0.084 - 0.075i$	$-0.082 - 0.026i$	$-0.091 - 0.007i$
		$[-0.121 - 0.071i]$	$[-0.130 - 0.053i]$
2-loop	$0.099 - 0.112i$	$-0.072 - 0.036i$	$-0.090 - 0.013i$
		$[-0.107 - 0.114i]$	$[-0.125 - 0.091i]$
Borel sum	$0.094 - 0.168i$	$-0.062 - 0.031i$	$-0.088 - 0.012i$
		$[-0.070 - 0.115i]$	$[-0.096 - 0.096i]$
Ambiguity	$0.058 - 0.008i$	$0.0004\ \ [0.002]$	$-0.0004\ \ [0.001]$

Table 2: Numerical results for the combinations of a_i parameters entering the $B \to \pi\pi$ decay amplitudes. Input parameters are $f_\pi = 131\,\mathrm{MeV}$, $f_B = 180\,\mathrm{MeV}$, $F_0^{B\to\pi}(0) = 0.28$, $\lambda_B = 350\,\mathrm{MeV}$, $m_b = 4.2\,\mathrm{GeV}$, $m_c = 1.3\,\mathrm{GeV}$, and $r_\chi = 1.197$. Perturbative results refer to the scale $\mu = m_b$ in the $\overline{\mathrm{MS}}$ scheme except for the hard-spectator contribution to a_2, where the scale is $\mu = \sqrt{\Lambda_h m_b}$ with $\Lambda_h = 0.5\,\mathrm{GeV}$ [3]. Borel resummed results are scale independent. Values given in brackets refer to the strict large-β_0 limit.

rows show the results obtained in the large-β_0 limit, at one-loop and two-loop order, and from a numerical evaluation of the Borel integrals. The last row contains the renormalon ambiguity. For the penguin coefficients, the default entries refer to the modified large-β_0 limit, whereas results obtained in the strict large-β_0 limit are given in square brackets. The only sizeable renormalon ambiguity arises from the hard-spectator term, which contributes to the real part of a_2 and corresponds to a first-order power correction from soft gluon exchange with the spectator quark. The ambiguity in the penguin coefficients is of order $(\Lambda_{\mathrm{QCD}}/m_b)^2$ and numerically very small. Based on our renormalon analysis, we thus find no evidence for a large dynamical enhancement of penguin amplitudes, in contrast with some recent conjectures [20, 21].

In regards to the numerical significance of higher-order perturbative corrections, the most striking effect is the large increase of the imaginary part of the coefficient a_2 as one goes to higher orders in perturbation theory. An analysis using momentum distribution functions shows that the imaginary part of the hard-scattering kernel contributing to a_2 is dominated by contributions from scales of order a GeV, indicating that it can receive large higher-order corrections [6]. The corresponding strong-interaction phase may therefore be larger than that obtained from the NLO analysis.

In the strict large-β_0 limit (shown by the numbers in brackets), the penguin coefficients a_4^p and a_6^p mirror the behavior of a_2. On the other hand, when the physical value $n_f = 5$ is used as in (3.25), the phases are reasonably close to those at NLO and do not receive large higher-order corrections. This shows that after the replacement in (3.25) there is a strong cancellation between the different terms in (3.14), which is maintained at higher orders. While the replacement in (3.25) was motivated on physical grounds, it is an open and important question whether such a delicate cancellation will persist once the full set of NNLO corrections is included in the theory of QCD factorization.

5. Phenomenological applications

This section addresses the effects of renormalon ambiguities and higher-order corrections on the phenomenology of B decays into two light pseudoscalar mesons. The results of section 4 have shown that the most notable effects are the ambiguity in the hard-spectator term and an increase in the imaginary parts of the kernels at higher orders in perturbation theory. We limit our phenomenological analysis to quantities that highlight these effects in a transparent way.

The $B^\pm \to \pi^\pm \pi^0$ decay amplitudes depend strongly on a_2 (see (3.1)) and are therefore a good probe of the renormalon ambiguity in the hard-spectator term. Using the data in table 2, and taking $|V_{ub}/V_{cb}| = 0.085$ and $|V_{cb}| = 0.041$, we find for the CP-averaged branching ratio

$$10^6\,\mathrm{Br}(B^\pm \to \pi^\pm \pi^0) = (4.57;\ 4.73;\ 4.74 \pm 0.50)\,, \tag{5.1}$$

where the numbers in parentheses refer to one-loop order, two-loop order, and Borel-resummation, respectively. The NLO result obtained using QCD factorization is 4.80. (The central value quoted in [3] is about 10% larger due to the inclusion of electroweak penguin contributions.) The magnitude of the renormalon ambiguity may be compared with the error estimate given in [3], where an uncertainty of $^{+0.8}_{-0.4}\,(\text{pars.}) \pm 0.3\,(\text{power})$ was assigned to account for the effects of parameter variations and power corrections from higher-twist distribution amplitudes. Since this is the mode that is the most vulnerable to the ambiguity in a_2, we are led to the conclusion that no large power corrections from soft gluon exchange are present in $B \to \pi\pi$, πK decays.

The CP asymmetries of certain decays can be very sensitive to the imaginary parts of the hard-scattering kernels and show clearly the effects of Borel resummation. As examples, we discuss the asymmetries for $B \to \pi^\pm K^\mp$ and $B \to \pi^0 K^\mp$, which to first approximation are given by

$$\begin{aligned} A_{\mathrm{CP}}(\bar{B}^0 \to \pi^+ K^-) &\propto -\sin\gamma \cdot \mathrm{Im}(a_4^c + r_\chi\, a_6^c)\,, \\ A_{\mathrm{CP}}(B^- \to \pi^0 K^-) &\propto -\sin\gamma \cdot \mathrm{Im}\left(\frac{a_4^c + r_\chi\, a_6^c}{1 + R_{\pi K}\, a_2}\right), \end{aligned} \tag{5.2}$$

where $R_{\pi K} \simeq F_0^{B\to K}(0)/F_0^{B\to\pi}(0) \times f_\pi/f_K \approx 0.9$. Using asymptotic distribution amplitudes, the a_i parameters for $B \to \pi K$ decays are identical to those for $B \to \pi\pi$. In figure 2, results for the two CP asymmetries are shown both in the strict large-β_0 limit (right) and in the modified scheme discussed in section 3.4. In both schemes, the asymmetries increase strongly at higher orders in perturbation theory, such that the Borel-resummed results are almost a factor of two larger than the results found at one-loop order. The two-loop contributions appear to be the most important higher-order effects. This trend suggests that the asymmetries predicted from exact NLO calculations in [3] will become larger upon the inclusion of higher-order terms. (As concerns the magnitude of the CP asymmetries, we believe that the true values will be somewhere in between the two schemes.) It is also apparent from the figure that, in all cases, higher-order perturbative corrections are much more important than the power corrections suggested by the values of the renormalon ambiguities. This provides additional motivation for calculating the imaginary part of the

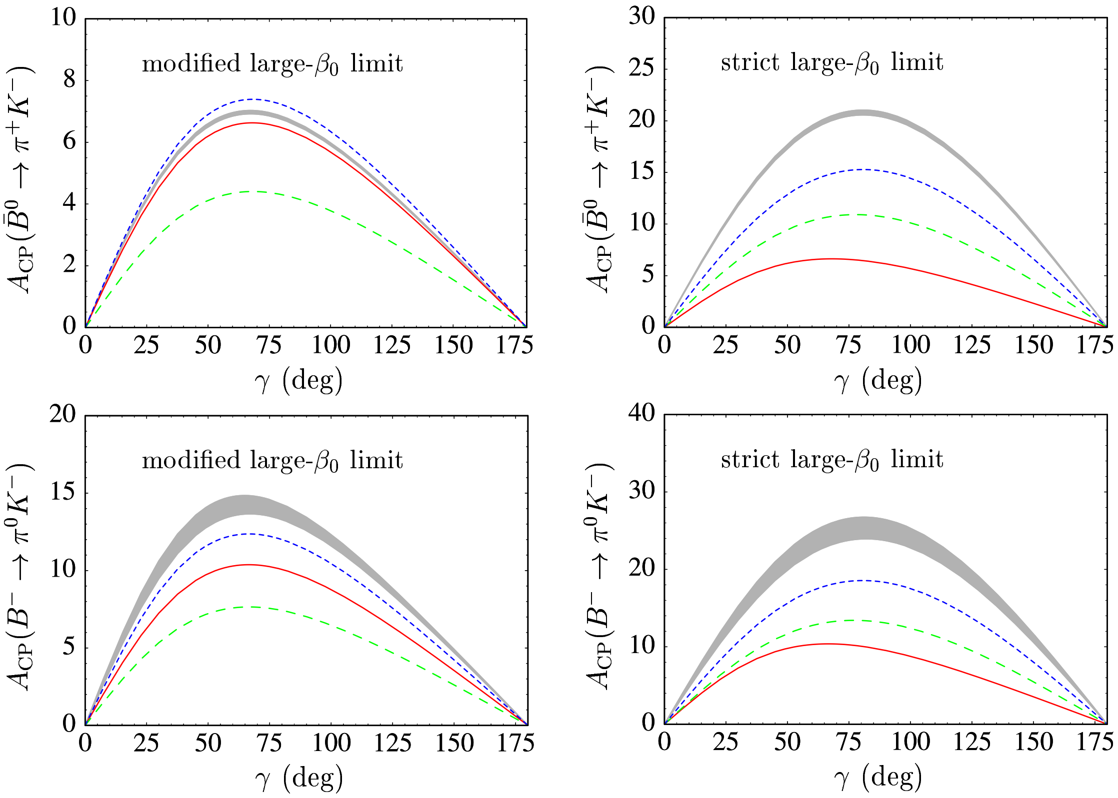

Figure 2: Predictions for the $B \to \pi^{\pm} K^{\mp}$ and $B^{\mp} \to \pi^0 K^{\mp}$ CP asymmetries in the strict (right) and the modified (left) large-β_0 limit. The different curves refer to the results at NLO (solid), one-loop order (long dashed), two-loop order (short dashed), and Borel-resummed (gray bands). For the Borel-resummed results, the width of the bands shows the renormalon ambiguity.

hard-scattering kernels at two-loop order. In fact, the two-loop imaginary parts (combined with the one-loop real parts) of the decay amplitudes are required for a consistent description of the CP asymmetries at NLO in RG-improved perturbation theory.

6. Conclusions

In this paper we have used the renormalon calculus and the large-β_0 limit of QCD to develop a toy model for higher-order corrections to the QCD factorization approach as applied to B decays into two light pseudoscalar mesons. We have presented novel results for the Wilson coefficients in the large-β_0 limit, and have used them to show that the RG invariance of the hard-scattering kernels entering the QCD factorization formula is preserved under Borel resummation. Both the Wilson coefficients and the coefficient functions a_i appearing in the amplitudes for $B \to \pi\pi$, πK decays simplify greatly in the large-β_0 limit, but still allow for a realistic study of power corrections and higher-order perturbative contributions to QCD factorization. We have presented exact expressions for the Borel transforms of the hard-scattering kernels in the factorization formula, from which is it possible to derive the terms

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of order $\beta_0^{n-1}\alpha_s^n$ in the perturbative expansions of the kernels. We have analysed the power corrections from soft gluon exchange and obtained a simple model for the first and second-order corrections to the parameters a_i. The only numerically significant power correction arises from soft gluon exchange with the spectator quark, which introduces an error in the CP-averaged $B^\pm \to \pi^\pm\pi^0$ branching ratio of about ten percent. This is roughly the same as the errors associated with the hadronic uncertainties and power corrections that were estimated in [3]. The power corrections to the real part of the penguin coefficients are of order $(\Lambda_{\rm QCD}/m_b)^2$, and those to the imaginary part are zero within the large-β_0 limit, which offers numerical evidence against models proposing dynamically enhanced penguins.

A recurring result of our study is that the imaginary parts of the hard-scattering kernels may be larger than expected from previous applications of QCD factorization. A calculation of the imaginary part of the kernels at two-loop order would be highly desirable, since it would give an answer to the question of whether there are large multi-loop contributions to these decays that are not captured in the large-β_0 limit.

Acknowledgments

We would like to thank Thomas Becher and Martin Beneke for useful discussions. This work was supported in part by the National Science Foundation.

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